Cambridge IGCSE™

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2024**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

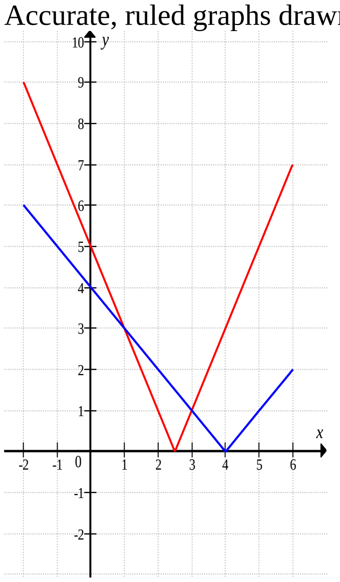
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$y-5=-2(x-3)$ oe	**3**	**M1** for midpoint $\left(\frac{5+1}{2},\frac{6+4}{2}\right)$ or $(3,5)$ **M1** for $m_{\perp}=\frac{-1}{\frac{1}{2}}$ oe or -2
	$\frac{121}{4}$ or 30.25 oe cao	**2**	**B1 FT** for x-intercept (5.5, 0) and y-intercept (0, 11) soi; **FT** *their* perpendicular bisector providing **M1 M1** awarded

Question	Answer	Marks	Partial Marks
2	Uses $b^2 - 4ac$ correctly: $(2k - 1)^2 - 4(k)(k + 1)$ [*0 where * is any inequality sign or =]	**M1**	
	Simplifies to $-8k + 1$[*0]	**A1**	
	Critical Value: $k = \frac{1}{8}$ soi	**M1**	**FT** *their* $ak + b$ where a and b are constants
	$k > \frac{1}{8}$ mark final answer	**A1**	
3(a)	Accurate, ruled graphs drawn 	**4**	**M1** for $y = \lvert 4 - x \rvert$: $\vee$ shape with vertex at (4, 0) **A1** Correct graph with y-intercept at (0, 4) **M1** for $y = \lvert 2x - 5 \rvert$: $\vee$ shape with vertex at (2.5, 0) **A1** Correct graph with y-intercept at (0, 5)
3(b)	$x \le 1$, $x \ge 3$ final answer	**2**	**FT** *their* **(a)** providing at least **M1** awarded and a pair of V-shaped graphs attempted **B1** for exactly two correct critical values or **B1 FT** for exactly two correct FT critical values soi, **FT** *their* **(a)** providing at least **M1** awarded and a pair of V-shaped graphs attempted
4(a)	$\frac{105}{8}$ isw or 13.125 oe	**2**	**B1** for ${}^{10}C_4 (x^2)^6 \left(-\frac{1}{2x^3}\right)^4$ oe

Question	Answer	Marks	Partial Marks
4(b)(i)	$1+4(2\sqrt{2})+6(2\sqrt{2})^2+4(2\sqrt{2})^3+(2\sqrt{2})^4$ soi or $1+4(-2\sqrt{2})+6(-2\sqrt{2})^2+4(-2\sqrt{2})^3$ $+(-2\sqrt{2})^4$ soi	**M1**	
	$1+8\sqrt{2}+48+64\sqrt{2}+64$ or $1-8\sqrt{2}+48-64\sqrt{2}+64$	**A1**	
	Correct difference stated or clearly implied $1+8\sqrt{2}+48+64\sqrt{2}+64-$ $(1-8\sqrt{2}+48-64\sqrt{2}+64)$	**M1**	**dep** on sight of correct expansions with numerical coefficients
	$144\sqrt{2}$ nfww	**A1**	
4(b)(ii)	$\frac{(their\ k)\sqrt{2}}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}=$ $\frac{(their\ k)\sqrt{2}-2their\ k}{-1}$ oe simplified to 2(*their k*) – (*their k*)$\sqrt{2}$ mark final answer	**2**	**STRICT FT** of *their* integer value of *k* **B1 STRICT FT** of *their* integer value of *k* for $\frac{their\ k\sqrt{2}}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}$
5(a)(i)	$\sec^2 x+2\tan^2 x$ oe and correct completion to $3\tan^2 x+1$ nfww	**2**	**M1** for use of a relationship to form a convincing correct statement from which the answer can be easily determined e.g. $\sec^2 x+\frac{2\sin^2 x}{\cos^2 x}$ or $\frac{1}{\cos^2 x}+2\tan^2 x$
5(a)(ii)	$\tan x=[\pm]1$ soi	**M1**	**FT** $\tan x=[\pm]\sqrt{\frac{4-their1}{their3}}$ providing $\frac{4-their1}{their3}>0$
	$[x=]\frac{\pi}{4},-\frac{\pi}{4}$ or ±0.785[39…] nfww and no other solutions	**A2**	**A1** for each, ignoring extra solutions

Question	Answer	Marks	Partial Marks
5(a)(iii)	$f'(x) = 6\tan x \sec^2 x$ oe	**M2**	**FT** $2(\textit{their}\,3)\tan x \sec^2 x$ **M1** for $f'(x) = k\tan x \sec^2 x$ where $k \neq 2\textit{their}\,3$
	$\left[f'\left(\frac{\pi}{4}\right)=\right]$ 12; $\left[f'\left(-\frac{\pi}{4}\right)=\right]$ – 12 nfww	**A2**	**A1** for each nfww
5(b)	Correct use of $\sin^2\theta + \cos^2\theta = 1$ to form a 3-term quadratic in $\sin\theta$ in solvable form $50\sin^2\theta + 5\sin\theta - 3[=0]$ oe	**M1**	Condone one sign or arithmetic error in rearrangement
	Solves or factorises *their* 3-term quadratic in $\sin\theta$ e.g. $(10\sin\theta + 3)(5\sin\theta - 1)$ [= 0]	**M1**	**FT** *their* 3-term quadratic in $\sin\theta$
	$\sin\theta = -0.3$ $\sin\theta = 0.2$ soi	**A1**	
	11.5 or 11.53[69...] 168.5 or 168.46[30...] 197.5 or 197.45[76...] 342.5 or 342.54[23...]	**A2**	with no extras in range **A1** for any two correct angles, ignoring extras
6(a)	$(2x + 1)(x - 3)(x + 1)$ nfww	**M1**	
	Correct method leading to $x = -\frac{1}{2}$, $x = 3$, $x = -1$	**A1**	
6(b)(i)	$[p'(x) =]\, 3x^2 + 2ax + b\, [= 0]$	**B1**	
	$3\left(\frac{4}{3}\right)^2 + 2a\left(\frac{4}{3}\right) + b = 0$	**B1**	OR forms the product $(3x - 4)(x - 2) = 0$
	$3(2)^2 + 2a(2) + b = 0$	**B1**	OR multiplies out to find $3x^2 - 10x + 8 = 0$
	Solves to find the value of one unknown	**M1**	**FT** *their* linear equations in a and b oe OR compares coefficients to state a value of a or b
	$a = -5$, $b = 8$	**A1**	
	[p(1)=] $1 + a + b + c = -5$ oe, soi	**M1**	
	$[1 - 5 + 8 + c = -5]$ $c = -9$	**A1**	

Question	Answer	Marks	Partial Marks
6(b)(ii)	$\left[p''(x) =\right] 6x + 2(\textit{their } a)$ soi	**M1**	**FT** *their a*
	6(2) – 10 = 2 > 0 [therefore minimum]	**A1**	
7(a)	$\frac{1}{2}\times 9^2 \times \theta - \frac{1}{2}\times 5^2 \times \theta = 4\pi$ oe, soi	**M2**	**M1** for $\frac{1}{2}\times 9^2 \times \theta$ or $\frac{1}{2}\times 5^2 \times \theta$ oe, soi
	$\theta = \frac{\pi}{7}$ oe or 0.449 or 0.4487 to 0.4488	**A1**	
7(b)	[Arc AD =] $\frac{5\pi}{7}$	**2**	**M1** for [Arc AD =] $5 \times \textit{their } \frac{\pi}{7}$ **FT** any stated value of θ from **(a)**
	[AC =] 4.991[27...] rot to 4 or more sf	**2**	**M1** for [AC^2 =] $9^2 + 5^2 - 2(9)(5)\cos\left(\textit{their } \frac{\pi}{7}\right)$ **FT** *their* θ providing $0 < \theta < \frac{\pi}{2}$
	11.2 or 11.23[526...] rot to 4 or more sf	**A1**	
8	$\int \cos\left(4x - \frac{\pi}{4}\right)dx = \frac{1}{4}\sin\left(4x - \frac{\pi}{4}\right)(+c)$	**B2**	**B1** for $k\sin\left(4x - \frac{\pi}{4}\right)$ where $k > 0$ or $k = -\frac{1}{4}$
	$\frac{3}{4} = \frac{1}{4}\sin\left(4\left(\frac{3\pi}{16}\right) - \frac{\pi}{4}\right) + c$	**M1**	**FT** *their k* providing **B1** awarded
	$-\frac{1}{16}\cos\left(4x - \frac{\pi}{4}\right) + \frac{1}{2}x \quad (+A)$	**2**	**M1 FT** for $m\cos\left(4x - \frac{\pi}{4}\right) + \left(\textit{their } \frac{1}{2}\right)x \quad (+A)$ **FT** $\textit{their } k \sin\left(4x - \frac{\pi}{4}\right) + \textit{their } c$ providing at least **B1 M1** awarded
	$y = -\frac{1}{16}\cos\left(4x - \frac{\pi}{4}\right) + \frac{1}{2}x + \frac{5\pi}{32}$ oe, cao	**2**	**M1 FT** for $\frac{\pi}{4} = -\frac{1}{16}\cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + \frac{1}{2}\left(\frac{3\pi}{16}\right) + A$ **FT** $(\textit{their } m)\cos\left(4x - \frac{\pi}{4}\right) + \left(\textit{their } \frac{1}{2}\right)x + A$ providing previous **M1** awarded

Question	Answer	Marks	Partial Marks
9	$\frac{dy}{dx} = -(3x-1)^{-2} \times 3$	M2	**M1** for $\frac{dy}{dx} = -k(3x-1)^{-2}$ where $k > 0$
	[When $x = 1$] $\frac{dy}{dx} = -\frac{3}{4}$ and $y = \frac{9}{2}$	A1	**FT** *their* $\frac{dy}{dx}$ providing **M1** has been awarded
	Equation of tangent: $y - \frac{9}{2} = -\frac{3}{4}(x-1)$ oe isw	M1	**FT** the value of *their* $\frac{dy}{dx}$ at $x = 1$ and *their* y
	$B(7, 0)$ oe	A1	
	Area of triangle: $\frac{1}{2} \times \frac{9}{2} \times ((their 7) - 1)$ or $\frac{27}{2}$ nfww or $-\frac{3}{8}(49) + \frac{21}{4}(7) - \left(-\frac{3}{8} + \frac{21}{4}\right)$	M1	**FT** *their* 7 and *their* $-0.75x + 5.25$ of the form $mx + c$ if needed
	[Area under curve = F(x) =] $\left[4x + \frac{1}{3}\ln(3x-1)\right]_1^9$ oe	B2	**B1** for $\int \frac{1}{3x-1}\,dx = k\ln(3x-1)$ or $\frac{1}{3}\ln 3x - 1$
	Correct and actioned plan e.g. F(9) – F(1) – *their* $\frac{27}{2}$	M1	**dep** on at least previous **B1** and correct plan or correct **FT** area of triangle oe
	$18\frac{1}{2} + \frac{1}{3}\ln 13$ or 19.4 or 19.35[49...]	A1	
10	$\overrightarrow{OP} = \lambda(\mathbf{a} + \mathbf{c})$ oe	B1	
	$\overrightarrow{OP} = \mathbf{a} + \mu\left(-\mathbf{a} + \frac{2}{5}\mathbf{c}\right)$ oe	B2	**B1** for $\overrightarrow{OP} = \mathbf{a} + \mu\left(-\mathbf{a} + \left(their\ \frac{2}{5}\right)\mathbf{c}\right)$
	Equates components at least once: $\lambda = 1 - \mu$ or $\lambda = \frac{2}{5}\mu$	M1	**FT** providing at least **B1** awarded
	Equates components: $\lambda = 1 - \mu$ and $\lambda = \frac{2}{5}\mu$	A1	
	$\mu = \frac{5}{7}$ $\lambda = \frac{2}{7}$ and $DP : PA = 2 : 5 = OP : PB$ oe	A2	**A1** for $\mu = \frac{5}{7}$ or $\lambda = \frac{2}{7}$